Thomas Kesselheim
Anna Heuser

June 8, 2026
Due date: June 15, 2026 at 8:00 p.m.

Algorithmic Game Theory

Summer Term 2026

Exercise Set 6

If you would like to submit your solutions for this problem set, please send them via email to aheuser1@uni-bonn.de by Monday evening. Submitting solutions in groups is also possible.

If you would like to present one of your solutions in class, please use the following link to book a presentation slot by Monday evening:
https://terminplaner6.dfn.de/b/8ea670502aa93dfa85aea3e5153d226c-1787955
A short meeting to discuss your solution is mandatory before presenting it in class. To book a time slot for this meeting, please use the following link by Monday evening as well:
https://terminplaner6.dfn.de/b/2062a052929bfd667ae801caf1a7fe59-1787947

Exercise 1:
Consider a *Knapsack Auction* which is defined the following way. Each bidder i has a publicly known weight w_i and a private value v_i. A feasible outcome is any set S of bidders such that $\sum_{i \in S} w_i \leq W$ holds for a fixed bound W. Furthermore, we assume that $0 \leq w_i \leq W$ for all bidder i.
The following algorithm yields a 2-approximation:

- Sort and renumber the bidders such that $\frac{b_1}{w_1} \geq \frac{b_2}{w_2} \geq \dots \frac{b_n}{w_n}$. Let k be the largest integer such that $\sum_{i=1}^{k} w_i \leq W$ and set $S_1 = \{1, \dots, k\}$.
- Let i^* be the bidder with the maximum bid b_i among all bidders and set $S_2 = \{i^*\}$.
- Return the better solution of S_1 and S_2.

(a) Show that the given algorithm is monotone.

(b) Show that the algorithm yields a 2-approximation.

Exercise 2:
As seen in lecture 13, let $f\colon V \to X$ be a function that maximizes declared welfare, i.e., $f(b) \in \arg\max_{x \in X} \sum_i b_i(x)$ for all $b \in V$. For each i, let h_i be an arbitrary function $b_{-i} \mapsto h_i(b_{-i})$ which does not depend on b_i. We define a mechanism $\mathcal{M} = (f, p)$ by setting

$$p_i(b) = h_i(b_{-i}) - \sum_{j \neq i} b_j(f(b)) \ .$$

Prove that $\mathcal{M}$ is a truthful mechanism.

Exercise 3:
Consider the following *Procurement Auction*. It's being attempted to buy a certain item. There are n vendors who are able to manufacture the wanted item. Vendor i incurs a cost of c_i for crafting the item. Now, the vendors are asked to state their costs for crafting the item and a vendor with lowest cost shall be chosen. The latter potentially gets a payment for it. The stated problem can be formalized by the general model of the lecture: Each vendor i is interpreted as a bidder who has negative valuation v_i, if he/she is chosen to craft the item, that is, $v_i(x) = -c_i$, if i is chosen in x.

(a) The results of the lecture concerning VCG are applicable in this situation. Make use of them in order to state a truthful mechanism.

(b) Use your results from the previous exercise to make the mechanism individually rational.

Exercise 4:
We consider a single-item auction via a mechanism which follows the spirit of Lecture 14, Section 2: All bidders submit their bids b_i. Fix a price of p (may depend on b) for the item. Approach bidders in order $1, \ldots, n$. As we consider bidder i: if the item is not allocated yet, assign the item for a price of p if $b_i - p \geq 0$.

(a) If $b = v$, show that the social welfare obtained by this auction is at least

$$\max_i v_i \mathbb{1}_{\text{item not allocated}} + p\left(\mathbb{1}_{\text{item allocated}} - \mathbb{1}_{\text{item not allocated}}\right) .$$

(b) Use your result from (a) to set a price obtaining a social welfare of at least $\frac{1}{2} \max_i v_i$ if $b = v$.