Thomas Kesselheim

November 20, 2018
Due: lecture on November 27, 2018

Algorithms and Uncertainty

Winter Semester 2018/19

Exercise Set 5

Exercise 1: (4 Points)
Show that Stochastic Set Cover can be reduced to the deterministic problem. To this end, define a different universe of elements U', family of subsets $\mathcal{S}'$, and costs $(c'_{S'})_{S' \in \mathcal{S}'}$ appropriately. Any solution of this Set Cover instance then corresponds to a policy of the same cost.

Exercise 2: (4+4 Points)
The Minimum Multiway Cut problem on trees is defined as follows. One is given a tree $G = (V, E)$ with edge weights $(w_e)_{e \in E}$. Furthermore, one is given k pairs $(s_i, t_i) \in V \times V$. The task is to find a set $S \subseteq E$ such that for all i the vertices s_i and t_i are not connected in $(V, E \setminus S)$.
A known approximation algorithm for this problem uses the following linear program. Let P_i be the (unique) path from s_i to t_i.

$$\begin{aligned} \text{minimize } & \sum_{e \in E} w_e x_e & \\ \text{subject to } & \sum_{e \in P_i} x_e \geq 1 & \text{for } i = 1, \ldots, k \\ & x_e \geq 0 & \text{for all } e \in E \end{aligned}$$

The algorithm computes a solution of cost $2 \sum_{e \in E} w_e x_e^*$, where x^* is an optimal solution of this linear program.

(a) Write an LP relaxation for the stochastic multi-stage variant, in which only pairs (s_i, t_i) from an initially unknown subset $A \subseteq \{1, \ldots, k\}$ have to be separated. The first phase, edges can be eliminated at cost $(c_e^{\mathrm{I}})_{e \in E}$, in the second phase at cost $(c_e^{\mathrm{II}})_{e \in E}$.

(b) Use an optimal solution of the LP relaxation and the approximation algorithm for the deterministic problem to compute a 4-approximation of the optimal policy.

Exercise 3: (4 Points)
We consider the following modified version of the Boosted Sampling algorithm for stochastic Steiner tree from the lecture. The only difference is that it uses ℓ sets $S_1, \ldots, S_\ell$ in the first phase. Show that the approximation guarantee is $\max\{2(1 + \frac{\lambda}{\ell+1}), 2(\frac{\ell}{\lambda} + 1)\}$. It is enough to highlight the difference to the previous analysis.

Exercise 4 on the next page.

Exercise 4: (4 Points)

The Boosted Sampling approach can also be used for Two-Stage Stochastic Vertex Cover. For simplicity, we assume that $c_v^{\mathrm{I}} = 1$ and $c_v^{\mathrm{I}} = \lambda$ for all $v \in V$ and only consider the first stage.

We use the following algorithm: In the first stage, draw sets $E_1, \ldots, E_\lambda$ from the distribution. Let $F_0 \subseteq V$ be the endpoints of any inclusion-wise maximal matching on $E_1 \cup \ldots \cup E_\lambda$. Show that $\mathbf{E}[|F_0|]$ is upper-bounded by the twice the expected cost of an optimal policy.

Bonus: Complete the algorithm and analysis for the second stage.