MA-INF 1203 Discrete and Computational Geometry

Wintersemester 2019/20
Assignment 2

To be discussed: **22/23**. October 2019

1 Affine Hull

- For any set $X \subseteq \mathbb{R}^d$, let $\text{aff}_\cap(X)$ denote the intersection of all affine subspaces of $\mathbb{R}^d$ containing X.
- For any set $X \subseteq \mathbb{R}^d$, let $\text{aff}_c(X)$ denote the set of all affine combinations of points of X.

Prove that $\text{aff}_\cap(X) = \text{aff}_c(X)$.

2 Convex Hull

i) Prove that the set of all convex combinations of $x_1, \ldots, x_n \in \mathbb{R}^d$ is a convex set.

ii) Prove by induction on n that the set of all convex combinations of $x_1, \ldots, x_n \in \mathbb{R}^d$ is contained in the convex hull of the set $\{x_1, \ldots, x_n\}$.

iii) Prove that for any ~~closed~~ finite set X the convex hull $\text{conv}(X)$ is equal to the intersection of all closed halfspaces that contain X.

3 Translated copies

Let $K \subset \mathbb{R}^d$ be a convex set and let $C_1, \ldots, C_n \subseteq \mathbb{R}^d$, $n \geq d+1$, be convex sets such that the intersection of every $d+1$ of them contains a translated copy of K. Prove that the intersection of all sets C_i also contains a translated copy of K that is $\exists t \in \mathbb{R}^d : \{t + x \mid x \in K\} \subseteq \bigcap_{i=1}^{n} C_i$.