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Algorithmic Game Theory

Winter Term 2021/22

Tutorial Session - Week 8

Exercise 1:
In Lecture 13, we have used a greedy algorithm in order to get a 2-approximation for the edge weighted bipartite matching problem (we used it in the context of unit-demand combinatorial auctions).
Show that the solutions of the algorithm are monotone in each component. I.e., if e is an edge chosen by the algorithm, then e will be also chosen if its weight is raised provided that all other weights remain unchanged.

Exercise 2:
Consider a *Knapsack Auction* which is defined the following way. Each bidder i has a publicly known weight w_i and a private value v_i. A feasible outcome is any set S of bidders such that $\sum_{i \in S} w_i \leq W$ holds for a fixed bound W. Furthermore, we assume that $0 \leq w_i \leq W$ for all bidder i.
The following algorithm yields a 2-approximation:

- Sort and renumber the bidders such that $\frac{b_1}{w_1} \geq \frac{b_2}{w_2} \geq \ldots \frac{b_n}{w_n}$. Let k be the largest integer such that $\sum_{i=1}^{k} w_i \leq W$ and set $S_1 = \{1, \ldots, k\}$.
- Let i^* be the bidder with the maximum bid b_i among all bidders and set $S_2 = \{i^*\}$.
- Return the better solution of S_1 and S_2.

Show that the given algorithm is monotone and state a truthful mechanism with the aid of Myerson's Lemma.