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Due date: October 27, 2021 at 10:00 a.m.

Algorithmic Game Theory

Winter Term 2021/22

Exercise Set 2

If you want to hand in your solutions for this problem set, please send them via email to alexander.braun@uni-bonn.de *– make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to alexander.braun@uni-bonn.de *containing the **task** which you would like to present and in **which of the tutorials** you would like to do so. Deadline for the email is Wednesday, 10:00 am. Please note that the tasks will be allocated via a first-come-first-served procedure, so sending this email earlier than Wednesday morning is highly recommended.*

Exercise 1: (3+2 Points)

Consider the bimatrix game *Battle of the Sexes* given in Example 3.3 of the third lecture.

a) Compute a mixed Nash equilibrium by choosing probabilities for one player that will make the other player indifferent between his pure strategies (see Example 3.11).

b) Determine the probabilities of the mixed Nash equilibrium graphically by plotting the players' expected costs.

Exercise 2: (4 Points)

We define a strategy $s_i \in S_i$ of a normal-form cost-minimization game to be *strictly dominated*, if there exists a strategy s_i' such that $c_i(s_i', s_{-i}) < c_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$. Prove that for all mixed Nash equilibria σ, there is no player $i \in \mathcal{N}$ with a mixed strategy σ_i such that $\sigma_{i,s_i} > 0$ for a strictly dominated strategy $s_i \in S_i$.

Exercise 3: (3 Points)

Have a look at the proof of Nash's Theorem (4.3) in which normal-form payoff-maximization games are considered. Let $\mathcal{N} = \{1, \ldots, n\}$ and $S_i = \{1, \ldots, m_i\}$ for all $i \in \mathcal{N}$. The set of mixed states X can be considered as a subset of $\mathbb{R}^m$ with $m = \sum_{i=1}^{n} m_i$.

Show that X is convex and compact.