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Algorithmic Game Theory

Winter Term 2021/22

Exercise Set 3

Exercise 1: (3+4 Points)

Consider the local search problem *Positive Not-All-Equal kSat* (Pos-NAE-kSAT) from Tutorial 3, Task 1 which is defined the following way:

Instances: Propositional logic formula with n binary variables $x_1, \ldots, x_n$ that is described by m clauses $c_1, \ldots, c_m$. Each clause c_i has a weight w_i and consists of exactly k literals, which are all positive (i.e., the formula does not contain any negated variable $\overline{x}_i$).

Feasible solutions: Any variable assignment $s \in \{0, 1\}^n$

Objective function: Sum of weights of clauses c_i in which not all literals are mapped to the same value.

Neighbourhood: Assignments s and s' are *neighbouring* if they differ in the assignment of a single variable.

In Tutorial 3, Task 1, we already showed that Pos-NAE-kSAT is in PLS. Now:

(a) Show that Pos-NAE-2SAT $\leq_{PLS}$ MaxCut

(b) Show that Pos-NAE-3SAT $\leq_{PLS}$ Pos-NAE-2SAT

Exercise 2: (4 Points)

We define a Congestion Game to be *symmetric*, if $\Sigma_1 = \ldots = \Sigma_n$. Let $PNE_{\text{Cong. Game}}$ and $PNE_{\text{Sym. Cong. Game}}$ be the local search problems in PLS of finding a pure Nash equilibrium of a general or symmetric Congestion Games, respectively.

Show: $PNE_{\text{Cong. Game}} \leq_{PLS} PNE_{\text{Sym. Cong. Game}}$.

Hint: Add an auxiliary resource for each player with a suitable delay function.

Exercise 3: (3+3 Points)

We want to derive properties of the sets of correlated and coarse correlated equilibria.

(a) Show that the set of correlated equilibria of a cost-minimization game Γ is convex, i.e. for two correlated equilibria p, p' and $\lambda \in [0, 1]$, also $\lambda p + (1 - \lambda)p'$ is a correlated equilibrium.

(b) Show that every correlated equilibrium is also a coarse correlated equilibrium.