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Due date: December 15, 2021 at 10:00 a.m.

Algorithmic Game Theory

Winter Term 2021/22

Exercise Set 9

Exercise 1: (4 Points)
An *all-pay auction* is a single-item auction defined in almost the same manner as a first-price auction: Each bidder reports a bid $b_i \geq 0$. The bidder with the highest bid wins the item. However, every bidder must pay their own bid regardless of whether they win the item or not.

Be inspired by the steps of Section 2 in the notes of Lecture 15 to derive the symmetric Bayes-Nash equilibrium of an all-pay auction with n bidders and identical distributions.

Exercise 2: (4 Points)
Show that if a mechanism is (λ, μ)-smooth and players have the possibility to withdraw from the mechanism then $PoA_{\mathsf{CCE}} \leq \frac{\max\{1,\mu\}}{\lambda}$.

Exercise 3: (3 Points)
Recall the auction of k identical items from the previous exercise sets. Bidder i has value v_i if he/she gets one of the items, 0 otherwise.
We define a mechanism as follows: the bidders who reported the k highest bids win an item. Each of them has to pay their respective bids.
Show that if losers (i.e. bidders who do not get any item) pay their bids, this mechanism is $(\frac{1}{2}, 2)$-smooth.

Remark: Notice the difference concerning the losers' payments compared to Exercise 2 from the tutorials. Now, all bidders pay their respective bids.