

Algorithms and Uncertainty

Winter Term 2025/26

Exercise Set 1

If you would like to submit your solutions for this problem set, please send them via email to ahouser1@uni-bonn.de by Monday evening. Submitting solutions in groups is also possible.

If you would like to present one of your solutions in class, please use the following link to book a presentation slot by Monday evening:

<https://terminplaner6.dfn.de/b/e2b70fff6726bae347216fb02513695f-1422679>

A short meeting to discuss your solution is mandatory before presenting it in class. To book a time slot for this meeting, please use the following link by Monday evening as well:

<https://terminplaner6.dfn.de/b/acd325fb62822fb110d396dbbcab9beb-1415737>

Exercise 1: (1+3 Points)

Consider the following algorithm for the ski rental problem: Buy a pair of skis on your first skiing day for a price of B .

- (a) Give a sequence σ of skiing/non-skiing days such that $c(\text{ALG}(\sigma)) = c(\text{OPT}(\sigma))$.
- (b) Is there an $\alpha > 0$ such that this algorithm is strictly α -competitive? If yes, give a proof, otherwise a counterexample.

Exercise 2: (3 Points)

In an undirected graph $G = (V, E)$ a Vertex Cover is defined as a set $C \subseteq V$, where $e \cap C \neq \emptyset$ for all $e \in E$.

The Online Bipartite Vertex Cover problem is defined as follows: We are given a bipartite graph with vertices $V = L \cup R$. The vertices in L are offline which means they are present initially. Vertices in R are online and revealed one at a time together with their incident edges. Each vertex $v \in V$ has a cost c_v . At any point in time we need to maintain a Vertex Cover C on the present graph. We may only add but not remove vertices from C . The goal is to minimize the total cost $\sum_{v \in C} c_v$.

Consider the ski rental problem in a simplified version, i.e. we assume that every day is a skiing day but we do not know the number of days in advance. Show in a constructive way that the ski rental problem is a special case of the Online Bipartite Vertex Cover problem.

Exercise 3: (4 Points)

We want to show that the assumptions (completeness and triangular inequality) in the lecture concerning the online Steiner tree problem are indeed without loss of generality. Therefore, consider an α -competitive online algorithm for the online Steiner tree problem on complete graphs satisfying the triangular inequality (a.k.a. metric Steiner tree problem). Show how to convert this algorithm into an α -competitive one for the general online Steiner tree problem.