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November 3, 2025
Due: November 10, 2025 at 10:00 p.m.

Algorithms and Uncertainty

Winter Term 2025/26

Exercise Set 4

If you would like to submit your solutions for this problem set, please send them via email to aheuser1@uni-bonn.de by Monday evening. Submitting solutions in groups is also possible.

If you would like to present one of your solutions in class, please use the following link to book a presentation slot by Monday evening:
https://terminplaner6.dfn.de/b/9d94240d8784212258801d0db55746b6-1452156
A short meeting to discuss your solution is mandatory before presenting it in class. To book a time slot for this meeting, please use the following link by Monday evening as well:
https://terminplaner6.dfn.de/b/5a5bf7bee2335351abfa17635aeffb25-1452165

Exercise 1: (1+4 Points)

Consider the following randomized algorithm for Online Bipartite Matching:
Whenever a vertex $r \in R$ is revealed, let L_r be the set of currently unmatched neighbors of r. Then choose any $l \in L_r$ uniformly at random and match r to l.

(a) Explain the difference between this algorithm and the Ranking Algorithm from Lecture 7.

(b) We are given an instance of Online Bipartite Matching with n offline nodes $\ell_1, \dots, \ell_n$ and n online nodes that appear in order $r_1, \dots, r_n$. For every $i \in [n]$, r_i is connected to ℓ_i. Additionally for every $i \in [\frac{n}{2}]$, r_i is connected to every node in $\{\ell_{\frac{n}{2}+1}, \dots, \ell_n\}$. Show that the algorithm achieves an expected competitive ratio of at most $\frac{1}{2} + \frac{O(\log n)}{n}$ on this instance.

Exercise 2: (4 Points)

Let us consider a generalization of the version of Markov decision processes covered in the lecture. For every state $s \in \mathcal{S}$, only a subset of the actions $\mathcal{A}_s \subseteq \mathcal{A}$, $\mathcal{A}_s \neq \emptyset$, is available. Devise an algorithm that computes an optimal policy for a finite time horizon T, show its correctness, and give a bound on its running time.