November 10, 2025 Due: November 17, 2025 at 10:00 p.m.

## Algorithms and Uncertainty

Winter Term 2025/26 Exercise Set 5

If you would like to submit your solutions for this problem set, please send them via email to aheuser1@uni-bonn.de by Monday evening. Submitting solutions in groups is also possible.

If you would like to present one of your solutions in class, please use the following link to book a presentation slot by Monday evening:

https://terminplaner6.dfn.de/b/80bc2522fe4e6f738218c8d3874d0fe3-1472676

A short meeting to discuss your solution is mandatory before presenting it in class. To book a time slot for this meeting, please use the following link by Monday evening as well: https://terminplaner6.dfn.de/b/2f2e21bffd2e5859d30fbbe6bca66fb4-1472670

Exercise 1: (2 Points)

Consider the cost-minimization variant of the optimal stopping problem in which we know the prior distributions. In step i, we can stop the sequence at cost  $c_i$ . We have to stop the sequence at some point and want to minimize the cost for doing so.

Show that there is **no**  $\alpha < \infty$  such that for all distributions the optimal policy has cost at most  $\alpha \mathbf{E}[\min_i c_i]$ .

Exercise 2: (5 Points)

We consider the following stochastic decision problem: There are n boxes; box i contains a prize of 1 with probability  $q_i$  and is empty otherwise. The game ends when we have found a non-empty box. That is, the final prize is either 0 or 1. At each point in time, we can also decide to stop playing. We can open as many boxes as we like but opening box i costs  $c_i$ . The goal is to maximize the final prize minus the sum of opening costs. Find an optimal policy.

**Hint:** It can be useful to consider the cases n=1 and n=2 first.