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November 17, 2025
Due: November 24, 2025 at 10:00 p.m.

Algorithms and Uncertainty

Winter Term 2025/26

Exercise Set 6

If you would like to submit your solutions for this problem set, please send them via email to aheuser1@uni-bonn.de by Monday evening. Submitting solutions in groups is also possible.

If you would like to present one of your solutions in class, please use the following link to book a presentation slot by Monday evening:
https://terminplaner6.dfn.de/b/ae952500b4b7bf892ca6a204c685af85-1481369
A short meeting to discuss your solution is mandatory before presenting it in class. To book a time slot for this meeting, please use the following link by Monday evening as well:
https://terminplaner6.dfn.de/b/2cd6a431fc482674d8e51b584801ddd7-1481366

Exercise 1: (3 Points)

Consider the following algorithm for the Secretary Problem from Lecture 10: Do not select any of the first $\lceil \frac{n}{2} \rceil$ candidates. Afterwards, pick the first candidate whose value exceeds all previous ones. Observe that this corresponds to a threshold algorithm with $\tau = \lceil \frac{n}{2} \rceil$.
Do not use Theorem 10.1 to show that this algorithm selects the best value with probability at least $\frac{1}{4}$.

Exercise 2: (4 Points)

We consider a variation of the Secretary Problem and the threshold algorithm. Again an adversary defines values $v_1, \ldots, v_n$. But instead of just drawing a random permutation, every candidate i draws an arrival time θ_i uniformly at random from the interval $[0, 1]$. Then the candidates arrive in increasing order of their arrival times.
The threshold algorithm now looks as follows: Observe all elements until time τ, then select the first candidate after time τ which is better than all preceding ones.
Show that for $\tau = \frac{1}{e}$ the threshold algorithm selects the best candidate with probability at least $\frac{1}{e}$.

Exercise 3: (3 Points)

Consider a rover that operates on a slope and uses solar panels to recharge. It can be in one of three states: high, medium and low on the slope.
If it spins its wheels, the probability to climb the slope (from low to medium or from medium to high or from high to high) is 0.3. With probability 0.7 it stays where it is.
If it does not spin its wheels, the probability to slide down the slope to low is 0.4. It stays where it is with probability 0.6.
Spinning its wheels uses one unit of energy per time step. Being high or medium on the slope gains three units of energy per time step via the solar panels, while being low on the slope does not gain any energy per time step. The robot wants to gain as much energy as possible in a given time horizon T.
What is the optimal policy for the rover?