Thomas Kesselheim
Anna Heuser

November 24, 2025
Due: December 1, 2025 at 10:00 p.m.

Algorithms and Uncertainty

Winter Term 2025/26

Exercise Set 7

If you would like to present one of your solutions in class, please use the following link to book a presentation slot by Monday evening:
https://terminplaner6.dfn.de/b/9cf56e7633cfd96b23b90d4de1dc952e-1496450
A short meeting to discuss your solution is mandatory before presenting it in class. To book a time slot for this meeting, please use the following link by Monday evening as well:
https://terminplaner6.dfn.de/b/72e4c1910cd8e8c59ad2770d448ec9a3-1496445

Exercise 1: (1+4 Points)

Consider the Pandora's Box problem from Lecture 12 but this time we are allowed to keep up to ℓ prizes instead of only one.

a) Define the fair-cap policy for this problem.

b) Show that the fair-cap policy is optimal.

Exercise 2: (3+4 Points)

In order to generalize the Pandora's Box setup from the lecture, suppose we would like to match people $i \in [n]$ to boxes $j \in [m]$ (each person can take at most one prize home). We know that person i's value v_{ij} for the prize in box j is independently drawn from a distribution $\mathcal{D}_{ij}$, but it costs c_{ij} to inspect the exact value of the box v_{ij}. Consider A_{ij}, I_{ij}, σ_{ij}, κ_{ij} and b_{ij} to be the corresponding generalizations of the variables introduced in the lecture.

(a) Show that for any policy π, the expected value is given by

$$V(\pi) = \sum_{i,j} \mathbf{E}\left[A_{ij}\kappa_{ij} - (I_{ij} - A_{ij})b_{ij}\right] \ .$$

(b) Consider the following generalized policy: Inspect the value of person i for item j v_{ij} in decreasing order of caps σ_{ij}. Every time, the highest observed value so far exceeds the largest remaining cap, i.e.

$$v_{i^*j^*} = \max_{\text{inspected } (i,j)} v_{ij} \quad > \quad \max_{\text{not inspected } (i,j)} \sigma_{ij},$$

we irrevocably match person i^* to box j^* and remove all incident edges to i^* and j^* from the graph.

Show that this policy achieves a value which is at least half the value of the optimal policy.
You can use without a proof that the greedy algorithm for bipartite matching which adds edges in decreasing order of weights to the matching achieves at least half of the optimal max-weighted matching in any bipartite graph.