## Algorithms and Uncertainty

Winter Term 2025/26 Tutorial Session - Week 2

If you do not know each other yet, each of you could start with a very quick introduction: What's your name? Do you study Computer Science or maybe something else (Maths, Economics,...)? Do you have any prior knowledge in Algorithms and Uncertainty? Afterwards, please discuss the exercises on this sheet. Note that you should see this also as a chance to talk about definitions, proof ideas and techniques used in the lecture in addition to only working out a formal solution for the tasks. If you do not know a definition or theorem by hard, feel free to open the lecture notes and have a look.

## Exercise 1:

Consider the following Set Cover instance:  $U = \{1, 2, 3\}$  and  $S = \{A, B, C\}$  with  $A = \{1, 2\}$ ,  $B = \{1, 3\}$ ,  $C = \{2, 3\}$ ,  $c_A = c_B = 3$ ,  $c_C = 4$ .

- (a) Give an optimal integral solution.
- (b) Give a fractional primal solution of cost at most 5.
- (c) Give a dual solution of value at least 5.
- (d) Use your solution of (c) to show optimality of your solution of (b). To this end, sum up the primal constraints in a suitable way. (Your solution should be in the spirit of proof of weak duality but not use the statement of the lemma itself.)