Thomas Kesselheim
Anna Heuser

November 26/27, 2025

# Algorithms and Uncertainty

Winter Term 2025/26

Tutorial Session - Week 7

**Exercise 1:**
Consider the following distribution for the prize of box $i$: the prize $v_i$ is equal to $w_i$ with probability $q_i$ and is 0 else. Assume that $\mathbb{E}[v_i] \geq c_i \geq 0$. Compute the fair cap.

**Exercise 2:**
Consider the minimization variant of Pandora's Box. We have $n$ boxes. Each of the boxes contains an item of certain weight. We may open as many boxes as we like, however opening box $i$ costs a certain amount. We have to take home one item and we need to open the box before taking the item. We may adapt our choices depending on what we find in the boxes that we open.
More formally, box i contains an item of weight $w_i$. We don't know $w_i$ but only its distribution until we open the box. Opening box $i$ costs $c_i$. The final weight is given as

$$\min_{i:\text{box } i \text{ opened}} w_i + \sum_{i:\text{box } i \text{ opened}} c_i.$$

We define the fair cap $\sigma_i$ such that $\mathbb{E}[b_i] + c_i = 0$, where $b_i = w_i - \kappa_i$ is the bonus and $\kappa_i = \max\{\sigma_i, w_i\}$ is the capped value.
Adapt the *fair cap* policy from the lecture appropriately and show that it is optimal for the minimization variant of Pandora's box.
You can use, without proof, that the value of any policy is again given by

$$V(\pi) = \mathbf{E}\left[\sum_i A_i \kappa_i - (I_i - A_i) b_i\right],$$

where $I_i$ indicates if the policy opens box $i$ and $A_i$ indicates, if the policy accepts the value in box $i$.