

Discrete and Computational Geometry

Winter semester 2026/2027

Assignment 1

Problem 1:

(6 Points)

Show that any set of $d + 2$ points in $\mathbb{R}^d$ is affinely dependent.

Hint: Consider the points $q_i = p_i - p_1$ for $i = 2, \dots, d + 2$ and use that they are linearly dependent.

Problem 2:

(3+3 Points)

Compute a Radon point for the following sets A_1 and A_2 . Derive the point using the steps in the proof of Radon's lemma.

- a) $A_1 = \{p_1, p_2, p_3, p_4\} \subset \mathbb{R}^2$ for the points $p_1 = (0, 1)$, $p_2 = (1, 0)$, $p_3 = (1, 2)$, $p_4 = (2, 1)$.
- b) $A_2 = \{p_5, p_6, p_7, p_8, p_9\} \subset \mathbb{R}^3$ for the points $p_5 = (0, 1, 2)$, $p_6 = (1, 0, 1)$, $p_7 = (2, 1, -1)$, $p_8 = (1, 1, -2)$, $p_9 = (-1, 2, 2)$.

Problem 3:

(4+4 Points)

Each set $X \subset \mathbb{R}^2$ of diameter at most 1 (i.e., any 2 points have distance at most 1) is contained in some disc of radius $\frac{1}{\sqrt{3}}$.

- a) Prove the statement for 3-element sets X .
- b) Prove the statement for all finite sets X .