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Problem Set 8

Problem 1

Let $n \in \mathbb{N}$, $m \in \{n, \ldots, n^2\}$, and $\phi \in \mathbb{N}$ be given. Find a minimum-cost flow network $G = (V, E, u, c)$ with $2n+2$ vertices, $m + 2n$ edges, and ϕ-perturbed edge costs on which the SSP algorithm needs m iterations for every realization of the edge costs. You may assume that for any nodes u and v the lengths of all possible u-v paths are pairwise distinct (Property 5.11).
Hint: This can be achieved such that there is a direct correspondence between the iterations and the edges that are neither connected to the source nor the sink.

Problem 2

Let $\phi \geq 5$ be given.

(a) Find a minimum-cost flow network $G = (V, E, u, c)$ with 4 vertices, containing a single source s and a single sink t, 5 edges, and ϕ-perturbed edge costs such that for one of the edges $e \in E$ the SSP algorithm will augment via a path containing e and via another path containing $\overleftarrow{e}$ for every realization of the edge costs.

(b) Let $\tilde{G}$ be the network created when the edge e from part (a) of the exercise is replaced by some single-source-single-sink-sub-network $G' = (V', E', u', c')$. What properties does G' need to have in order to guarantee that the SSP algorithm, when run on $\tilde{G}$, first augments via a series of paths that create a maximum flow in G' and then via another series of paths after that no flow is left on any of the edges of G'?

Problem 3

Let $n \in \mathbb{N}$, $m \in \{n, \ldots, n^2\}$, and $\phi \in \mathbb{N}$ with $\phi \in O(2^n)$ be given. Start with the instance from Problem 1 and use the result from Problem 2 to create an iterative method to create instances that contain $O(n)$ vertices, $O(m)$ edges, and ϕ-perturbed edge costs, on which the SSP algorithm augments via $\Omega(m \cdot \phi)$ many paths for every realization of the edge costs that satisfies Property 5.11.

How long can this iterative method be used? What are possible restricting parameters?

Problem 4

Prove that for given positive integers n, $m \in \{n, \ldots, n^2\}$, and $\phi \leq 2^n$ there exists a minimum-cost flow network with $O(n)$ nodes, $O(m)$ edges, and ϕ-perturbed edge costs on which the SSP algorithm requires $\Omega(m \cdot \min\{n, \phi\} \cdot \phi)$ augmentation steps.