

Selected Topics in Algorithmics, SS15
Exercise Sheet “1”: Quicksort and Trapezoidal
Decomposition
University of Bonn, Department of Computer Science I

- *Written solutions have to be prepared until **Tuesday 29th of April, 14:30 pm**. There is a letterbox in front of room E.01 in the LBH building.*
- *You may work in groups of at most two participants.*

Exercise 1: Probability Space (4 Points)

Consider a standard 52-card deck of poker cards. Assume we do not distinguish cards by their suits, i.e., cards with the same number are identical. We want to select 5 cards from the deck. Please define the probability space as follows.

1. Please describe the sample space Ω . (The outcome can be classified into 6 category, and each category has different number of elements.)
2. Please describe the family \mathcal{F} of events, e.g., the total number.
3. Please describe the probability function \Pr by illustrating the probability for the elements in the sample space. (just one element for each category)
4. Let X be the random variable representing the sum of 5 cards. Please compute the expectation of X .

Exercise 2: Average Complexity of Sorting (4 Points)

Given a set N of n real numbers, please analyze the average complexity for the following sorting algorithms over all the $n!$ permutation sequences of N .

- Insertion Sort
- Merge Sort
- Quick Sort (always select the first element)

Exercise 3: Vertical Trapezoidal Decomposition (4 Points)

Given a set N of n line segments with a total number k of intersection in the plane, let S_1, S_2, \dots, S_n be a random sequence of N , and let N^i be $\{S_1, S_2, \dots, S_i\}$. Please prove the following.

1. The vertical trapezoidal decomposition $H(N)$ of N has $O(n+k)$ trapezoids (faces) even if more than two line segments can intersect at the same point.
2. The expected number of trapezoids in $H(N^i)$ is $O(i + ki^2/n^2)$. (Hint: the expected number of intersections)